Bases for predicting gas-particle flows with particular reference to turbomachinery

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Advances in the understanding of particle dynamics and their trajectories and the resultant erosion of solid surfaces are reviewed. Particular attention is given to the calculation procedures used in the analysis of gas-particle flows, always with the intention of application to turbo-machines. One, two and three-dimensional analyses are considered and the limitations of each are discussed. As in the analysis of all real fluid flows, resort is made to empirical correlations of many phenomena and, even then, the predictions of particle trajectories are restricted. The correlation and restrictions are discussed where appropriate.

Key words: *turbomachines, particulate materials, erosion*

Particulate flows are found in many areas of engineering. Of most interest to the authors of this article are particulate flows in gas turbines. High velocity particles can impact on engine components, damaging their surfaces thus resulting in progressive degradation of engine performance and the reduction of the normal engine overhaul time and life.

Studies of the dynamics of multiphase systems have followed two approaches:

- studying the dynamics of a single particle and then attempting to extend the result to a multiple particle system in an analogous manner to the molecular theory;
- modifying the continuum mechanics of single-phase fluids to account for the presence of particles.

The first approach is advantageous for trajectory calculations and erosion studies and the second for the study of heat transfer, deposition, diffusion and corrosion problems. The behaviour of particle movements in fluid flows can be characterised in terms of collision or non-collision particles. Collision particles are rigid in nature and of sufficient kinetic energy to have rebound characteristics which cause erosion of the colliding surface; non-collision particles are deformable and can be chemically reactive, coalescing with each other or with the colliding surface causing deposition and diffusion.

High-velocity particulate impact with material surfaces has been treated as an engineering mechanics problem, assuming inelastic collision which results in erosion. Two different modes of erosion are distinguished for different classes of target materials, ductile and brittle. The ductile mode is typical of most metal targets in which the erosion mechanism might be one of cutting or micromachining with the target erosion loss being a maximum for an impingement angle between 20 and 30 degrees. The brittle mode is typical of glass and ceramic targets in which the erosion mechanism is one of fatigue of the target surface with the target erosion loss being a maximum for an impingement angle of 90 degrees. This paper reviews theoretical and experimental work on particulate flows and the ductile mode of erosion for application to gas turbines.

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Erosion mechanisms

According to Finnie[1], erosion loss per individual solid particle cutting through a ductile target surface is a function of the incident angle and is directly proportional to the kinetic energy of the particle. First Bitter[2] and then Neilson and Gilchrist[3] quantified Finnie's relationship to give, respectively:

$$Q = \frac{M(V \sin \alpha - K)^2}{2\varepsilon} \tag{1}$$

and

$$Q = \frac{M(V^2 \cos^2 \alpha - V_r^2)}{2\phi} + \frac{M(V \sin \alpha - K)^2}{2\varepsilon} \tag{2}$$

where Q is the volumetric erosion loss of target surface per impinging particle of mass M, velocity V and incident angle α, K is a constant related to the threshold velocity below which erosion cannot occur, V_r is the residual tangential component of particle velocity and ε and ϕ are the energies needed to remove unit volume of target material by repeated deformation and cutting wear respectively. Bitter's equation is based solely on an analysis of the brittle mode of erosion whereas Neilson and Gilchrist take account of both the brittle and ductile modes.

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Naturally occurring surface dusts and, therefore, particles in general, have different physical characteristics and various chemical properties. These variations coupled with target material properties and characteristics influence erosion rates and, therefore, the values of K, ε and ϕ. Smeltzer *et al*[4] conducted a number of erosion experiments on different target materials using dust particles. They concluded that metal target surface erosion is a minor by-product of the principal energy absorbing surface reaction of deformation; this was so whether the metal was annealed or fully heat treated. The amount of surface deformation appears to be most strongly dependent on the characteristics of the individual impacting particle rather than on the target material and environment. Smeltzer *et al* concluded that the erosion mechanism efficiency described by the 'ε' erosion factor is quite low ($1.14\times10^{-3}\leqslant\varepsilon\leqslant4.8\times10^{-3}$) for all their test dusts and target materials combinations; thus approximately 250 to 1000 parts by weight of dust were required to remove one part by weight of target material. Also they found that temperature had a variable effect on erosion; for a given metallic alloy target material at elevated temperature and high temperature gas, the erosion rate is greatest for impingement angles between 15 and 37 degrees and less at all other angles. Connors and Murphy[5] reported on erosion damage observed in the practical situation of gas turbines returned from service. The damaged regions were selective; for example, the compressor diffuser loss of surface material was due to low angle impact and in the case of the turbine stator blades the erosion damage was concentrated on one surface of each blade.

Notation

a	Radius
$2a_{jk}$	Mean diameter (see Eq (4))
$2a_M$	Mean mass diameter (see Eq (5))
c	Blade axial chord
C_1, C_2, C_3	Constants (see Eq (14))
C_D	Drag coefficient
C_f	Local skin friction coefficient
C_p	Specific heat
d	Particle diameter
D	Duct section hydraulic diameter
f_M	Mass distribution function
f_N	Number distribution function
F	Empirical multiplier to modify Stokes law
g	Acceleration due to gravity
h	Convective heat transfer coefficient
j	Integer (see Eq (4))
k	Integer (see Eq (4))
K	Constant
m_i	Mass per unit volume of the ith particle species
M	Mass of particle
n_i	Number of particle of mass m_i per unit volume
N	Number of particle size classes
p	Pressure
Q	Volumetric erosion loss of target surface per impinging particle
r	Length in the radial direction
$\dot{r}_i$	Velocity of the ith particle species in the radial direction
$\dot{r}_f$	Velocity of the fluid in the radial direction
Re	Reynolds number
St	Stokes number
St'	Modified Stokes number
t	Time
$\dot{t}_i$	Velocity of the ith particle species in the tangential direction
$\dot{t}_f$	Velocity of the fluid in the tangential direction
T	Particle temperature
T_f	Fluid temperature
T_0	Duct wall temperature
V	Particle velocity
V_r	Residual tangential component of particle velocity
V_1	Particle velocity before impaction
V_2	Particle velocity after impaction
V_{tank}	Tank volume
x	Length in the primary flow direction (cartesian coordinates)
$\dot{x}$	Particle velocity in the x direction
$\dot{x}_f$	Fluid velocity in the x direction
$\ddot{x}$	Particle acceleration in the x direction
α	Incident angle
α_{xi}	ith particle species acceleration in the x direction
α_{ri}	ith particle species acceleration in the r direction
β_1	Particle trajectory to surface angle before impaction
β_2	Particle trajectory to surface angle after impaction
Δt	Residence time of a particle in the tank
ε	Energy to remove unit volume of target material by repeated deformation
ε	Emissivity
ϕ	Energy to remove unit volume of target material by cutting wear
λ	Angularity factor
μ_f	Fluid viscosity
$\dot{\eta}_i$	Number flow rate of the ith particle species along the trajectory
ρ	Particle density
ρ_f	Fluid density
σ	Stefan–Boltzmann constant
τ_0	Wall shear stress
τ_r	Shear stress in the r direction
τ_x	Shear stress in the x direction

Subscripts

ℓ	Based on length
M	Based on mass
s	Based on surface area
v	Based on volume
vs	Based on volume–surface area

In view of the importance of erosion damage to engine performance and life, tests have been carried out on a number of possible engine component materials, both bare and coated. Typical of this work is that of Shoemaker and Shumate[6] who carried out tests on three substrate materials (N155, WI-52 and Inco 713C), bare, plasma spray coated, aluminised, chromised and plated from a fused salt bath. At elevated temperatures (1228 K) N155 was the most resistant substrate material to erosion damage when tested bare. The lowest weight loss factor was attained with N155 coated with the high density proprietry material Solar 2286-1277-28, while both aluminising and chromising offered too little protection to justify their use. The erosion tests of Shoemaker and Shumate, which are typical of many, showed that the use of selected coatings could extend engine service life by a factor of two in a severe dust environment. The choice of coatings, however, is very sensitive, depending in particular on the angle of impact of particles on the surface of interest.

Smeltzer *et al*[4] suggested using the average erosion loss per single particle impact, instead of the standard erosion factor ε and individual particle kinetic energy, as a better means of analysing erosion loss data. From the experimental work of Sheldon *et al*[7] the average erosion loss, that is the weight of target material removed per weight of impacting particle, was correlated with particle velocity by:

$$\text{Erosion loss} \propto V^{2.35} \tag{3}$$

Eq (3) was found to apply both to abrasive angular and spherical particles and the erosion loss was independent of particle size, except for particles of less than 100 μm. However, erosion loss is dependent on particle impact angle and this must also be accounted for in any model of erosion, as must the particle rebound characteristics. The numerical model used to predict the distribution and overall erosion rate in the experiments of Sheldon *et al* was an extension of the general two-phase flow model described by Crowe and Pratt[8] and Crowe *et al*[9] for gas-droplet flows.

Particle dynamics

In order to understand erosion mechanisms more fully, it is essential to predict particle dynamics and, therefore, particle trajectories in the fluid flows in machines. Whether using the early simplified analytical calculations of Gyamarthy[10] or the equally early computed predictions of Martlew[11] of particle trajectories or the latest three dimensional numerical-computational procedures, the particle forces and the fluid–particle interactions have to be modelled and therefore approximated.

Particle size

In general, solid particles are of irregular and haphazard shape and individual particles in a collection of particles chosen to be all of the same average size will not have the same rate of movement through a given uniform fluid flow. For modelling work, sand grains, for example, may be replaced by a collection of imaginary spheres of the same materials and of such a diameter, the mean diameter, that they will behave in the fluid flow in the same way as the average sand grain in the sample. In view of the irregular shape of particles, a multiplier known as a shape factor has to be applied to the mean diameter to determine the equivalent diameter used in calculating drag coefficient. In the case of desert sand, the shape factor is about 0.75 and typical particle sizes and their range of sizes (according to Soo[12]) are shown in Fig. 1. Thus it is necessary when considering particles to take account of their variations of size in a given size range in terms of either the variation of mass through a mass distribution function $f_M(b)$ or the variation of geometric size, say, equivalent diameter, through a number distribution function $f_N(b)$ where b is a measure of size. Cadle[13] defined the equivalent diameters for irregularly shaped particles depending on the transport phenomenon involved which, in summary, are given by:

$$(2a_{jk})^{j-k} = \int (2a)^j f_N \, da \Big/ \int (2a)^k f_N \, da \tag{4}$$

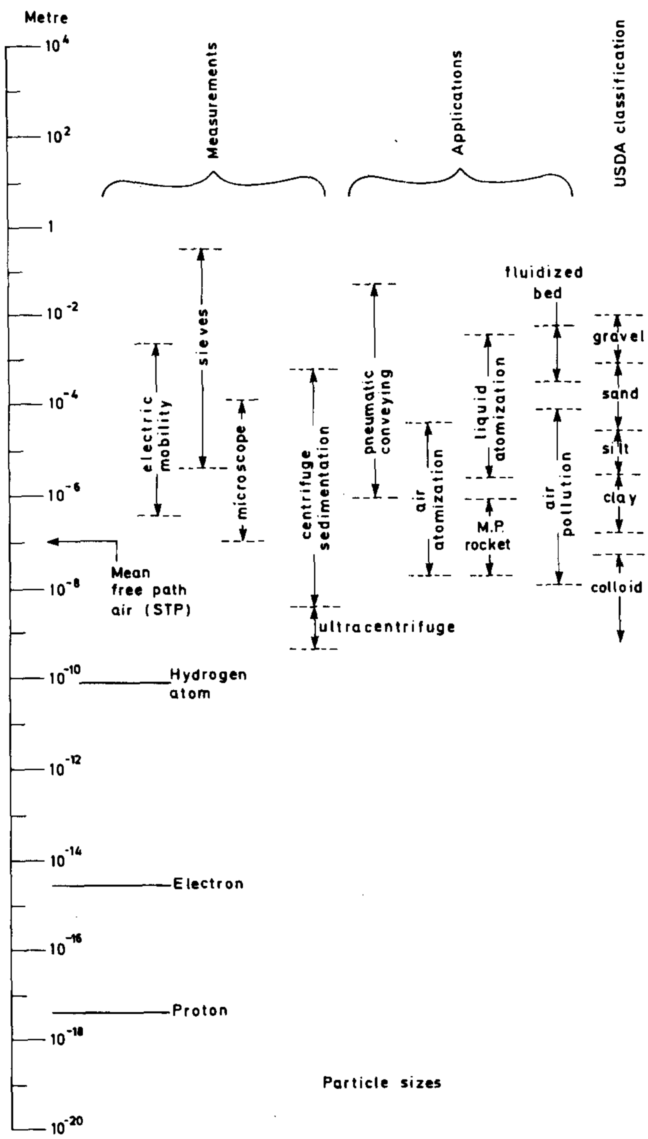

Fig 1 Magnitudes of particle sizes in multiphase systems in relation to other physical quantities (according to Soo[12])

The mean length diameter is given by $j=1$, $k=0$; mean surface area diameter by $j=2$, $k=0$; mean volume diameter by $j=3$, $k=0$; and mean volume-surface area diameter by $j=3$, $k=2$. Cadle also defined a mean mass diameter by:

$$2a_M = \int 2af_M \, da \Big/ \int f_M \, da \tag{5}$$

There are instances when the use of mean diameter breaks down; this is particularly so with fibrous particles whose shapes are far from spherical.

Pressure drop

For pipe flow, almost all investigators agree that loading ratio contributes to pressure losses though some, eg Tchen[14] and Richardson and McLennan[15], reported by Khan and Pei[16], have observed a decrease in pressure drop by addition of solid particles to gas flows. Thomas[17] attributed the decrease in pressure drop over a particular loading range to the non-Newtonian behaviour of the suspension. Pashatskiy *et al*[18] reported on the combined effect of particle size and loading ratio. When making pressure loss measurements in air for particles of 10 μm, 50 μm and 150 μm diameter and loading ratio of 0.5 to 4.0, they found that the pressure loss increased with increasing loading ratio and reducing particle size. Many workers have observed an increase in pressure drop in vertical pneumatic conveying with increase in particle Reynolds number, but Prem Chand and Gosh[19] were the first to use Froude number as a correlating parameter for pressure drops. Duckworth[20] suggested that the influence of Reynolds number was insignificant and only the Froude number is important. In rotating machinery, accelerations equivalent to many thousand 'g' can be encountered and if Froude number is an important correlation parameter in the earth's gravity, admittedly on large scale vertical equipment, it could be important in such machines. The mathematical modelling of gas–solids suspension flows as a continuum is a valid approach and, as Kramer and Depew[21] stated, 'this conclusion is warranted by the development of universal velocity profiles and a friction factor law which only depend on the particle size and mass loading ratio and is independent of Reynolds number'. The contribution of particle shape to pressure gradient was first studied by Jones and Allendorf[22,23], followed by Leung *et al*[24]. Although many workers have found this contribution to be negligible, other workers do take account of particle shape through an angularity correction to the drag coefficient and Stokes number in particle trajectory calculations.

Drag coefficient

Prem Chand and Gosh[19] were among the first to observe the dependence of pressure drop for particle laden flows in pipes on the ratio of drag coefficient to pipe friction. Brandt and Perini[25] considered three drag laws when predicting the effects of particle injection into a uniform flow. The three laws were:

- Stokes drag, $C_D Re = 24$, yielding a closed-form solution;
- standard drag law for a sphere based on available experimental data[26];
- an empirical drag curve accounting for Mach number effects[27].

Each of the three laws is accurate in circumstances specifically suitable to them. Rudinger[28] suggested that for flows where the particles represent less than one half of the mass of the mixture and the ratio of average particle spacing to particle diameter is ten or greater, the absence of interaction between particles appears plausible. Under these circumstances the standard drag law for a single sphere in steady flow applies; for particle Reynolds numbers below several hundred this approximates to:

$$C_D = \frac{24}{Re}\left(1 + \frac{Re^{2/3}}{6}\right) \tag{6}$$

At higher particle Reynolds numbers, deviations from the standard drag law occur with more rapid decrease of the drag coefficient with increasing Reynolds number than predicted by Eq (6)[12,29]. Rudinger reported from experiments in a shock tube that for $50 < Re < 300$:

$$C_D = \frac{6000}{Re^{1.7}} \tag{7}$$

and this relationship was found to be independent of particle concentration. Rudinger's finding may be unique to his experimental apparatus and experimental technique. For a similar Reynolds number range, $6 < Re < 400$, Ingebo[30] correlated his measurements for particle mass fractions of about one per cent by:

$$C_D = \frac{27}{Re^{0.84}} \tag{8}$$

It has become common practice today to use one law, in essence a modified Stokes drag using a multiplier dependent on particle Reynolds number, for all situations. Lord and Singh[31] used:

$$C_D = \frac{24F\lambda}{Re} \tag{9}$$

where λ is the angularity factor and F is an empirical expression, a polynomial in Re, allowing for the effect of the fluid-particle flow in modifying Stokes law. For example Langmuir and Blodgett[32] suggested:

$$F = 1 + 0.197Re^{0.63} + 0.00026Re^{1.38} \tag{10}$$

Tabakoff and Hamed[33] and Abdel Azim and Rashed[34] used Stokes law for the Reynolds number ranges $Re < 1$ and $Re < 0.1$ respectively for particulate flows in turbomachinery and, above these values of Re, both sets of authors modified the drag coefficient. Tabakoff and Hamed used:

$$C_D = \frac{24}{Re}\left(1 + \frac{3Re}{16}\right) \quad \text{for } 1 < Re < 4 \tag{11}$$

$$C_D = 0.342 + 21.942Re^{-0.178} \quad \text{for } 4 < Re < 2000 \tag{12}$$

$$C_D = 0.4 \quad \text{for } Re > 2000 \tag{13}$$

while Abdel Azim and Rashed used:

$$C_D = C_1 + \frac{C_2}{Re} + \frac{C_3}{Re^2} \tag{14}$$

where they quoted:

Re	C_1	C_2	C_3
0.1–1	3.69	22.73	0.093
1–10	0.36	8.80	−12.65
10–100	0.617	46.50	−116.67
100–1000	0.364	98.33	−2778.0
1000–5000	0.357	148.62	−47 500.0

Comparing Eqs (9) and (10) with (11), (12) and (13) and with (14) gives, for spheres, $C_D = 0.477$, 0.496 and either 0.460 or 0.458 at $Re = 1000$ and $C_D =$ 4.432, 4.542 and either 1.114 or 4.100 at $Re = 10$. Clearly the expression for C_D for $1 < Re < 10$ by Abdel Azim and Rashed is in error but otherwise the comparisons are fair.

When examining any fluid flow process with particulates present, the modelling process must account for the effects of loading ratio, density ratio and particle size and shape, Reynolds number and Froude number wherever appropriate and, of course, drag coefficient. In addition particle spin and lift forces may need to be considered as well as rebound characteristics from the flow field boundary surfaces. Ahmad and Goulas[35] calculated trajectories of spherical particles paying particular attention to particle behaviour on collision with a solid wall and including lift forces in their analysis. They found that when the angle of incidence between particle path and solid wall is below a critical value, the particle spins along the surface before leaving it. The effect of lift was found to be more pronounced with increasing values of gas velocity and decreasing values of particle size and density.

Particle trajectories

The analysis of gas-particle flows involves the conservation equations for mass, momentum and energy of gas and particles independently, but additional equations are needed which couple the mass, momentum and energy transfer between gas and particles.

The governing equations for one-dimensional flow of a single spherical particle of diameter d and density ρ in a gas of density ρ_f in the direction x of a gravitational acceleration 'g' ignoring gas container boundary wall effects, have long been understood. Apart from the weight of the sphere ($\pi d^3 \rho g/6$), the forces acting on the sphere are due to buoyancy ($-\pi d^3 \rho_f g/6$), the force associated with the kinetic energy of the fluid motion generated by doing work against a drag force ($-\pi d^3 \rho_f \ddot{x}/12$) and the force caused by fluid viscosity ($-\pi d^2 \rho_f \dot{x}|\dot{x}|C_D/8$) where $\dot{x}$ is the velocity and $\ddot{x}$ the acceleration of the particle in the x direction. It is reasonable to have omitted the force due to wave drag for low, subsonic speeds.

Thus the governing equation is:

$$\frac{\pi d^3 \rho \ddot{x}}{6} = \frac{\pi d^3 \rho g}{6} - \frac{\pi d^3 \rho_f g}{6} - \frac{\pi d^3 \rho_f \ddot{x}}{12} - \frac{\pi d^2 \rho_f \dot{x}|\dot{x}|C_D}{8} \tag{15}$$

which can be re-written as:

$$\frac{\ddot{x}}{g}\left(\frac{\rho_f}{2(\rho-\rho_f)}\right)\left(\frac{2\rho}{\rho_f}+1\right) + \frac{3C_D \rho_f \dot{x}|\dot{x}|}{4(\rho-\rho_f)dg} - 1 = 0 \tag{16}$$

Eq (16) shows that the acceleration of, and therefore resultant forces on, the particle, is a function of density ratio, Reynolds number through C_D and Froude number; note (Froude number)^2 is the ratio of inertial to gravitational forces. If gravitational forces are ignored, then Eq (15) is considerably reduced and shows that the characteristic parameter is (St/ReC_D) which for low Reynolds number flows reduces to a Stokes number only dependence. Eqs (15) or (16) can be solved by the Runge–Kutta method after substitution of suitable expressions for C_D.

One of the first analyses of one-dimensional steady motion of a gas–solid suspension through a duct was developed by Soo[12]. He suggested that the momentum equation for the particles, neglecting the volume occupied by the solid particles and relative acceleration and ignoring inter-particle forces, is given by:

$$\dot{x}\frac{d\dot{x}}{dx} = \frac{3}{4}\frac{\rho_f}{\rho d}(\dot{x}_f - \dot{x})|\dot{x}_f - \dot{x}|C_D \tag{17}$$

where $\dot{x}_f$ is the gas velocity in the x direction. Soo further suggested that the overall momentum equation with shear stress at the duct boundary being accounted for via skin friction is given by:

$$\rho_f \dot{x}_f \frac{d\dot{x}_f}{dx} + \sum \rho \dot{x}\frac{d\dot{x}}{dx} = -\frac{dp}{dx} - \frac{4\tau_0}{D} \tag{18}$$

where the summation sign refers to the sum for all particles in the duct section of interest, dp/dx is the pressure gradient, D is the duct section hydraulic diameter and the wall shear stress $\tau_0 = C_f(\rho_f(\dot{x}_f)^2/2)$, where C_f is the local skin friction coefficient ignoring the stress caused by impact of particles with the wall. Assuming that all particles in any one section of the duct at any one time are at the same temperature T, with gas temperature T_f and wall temperature T_0, the particle energy equation is:

$$\dot{x}\frac{dT}{dx} = \frac{6h}{\rho C_p d}(T_f - T) + \frac{6\varepsilon\sigma}{\rho C_p d}(T_0^4 - T^4) \tag{19}$$

where h is the convective heat transfer coefficient between gas and particle, C_p is the specific heat and ε the emissivity of the particle, σ is the Stefan–Boltzmann constant, and it has been assumed that the particle number density is low enough for diffuse radiation to be ignored. In deriving Eqs (17)–(19) Soo, like many other workers, ignored many particle, inter-particle and particle-duct boundary phenomena and suggested allowing for turbulence in the characteristic parameters.

One-dimensional analyses are extremely restrictive; the minimum should be two dimensional and, preferably, three dimensional. Crowe and Pratt[8] demonstrated a technique, the 'tank and tube' formulation from Gosman *et al*[36], to analyse subsonic two-dimensional particle laden gas flows. Gosman *et al* developed the 'tank and tube' finite-difference analogue for the solution of recirculating flow problems because it ensures that the conservation laws are obeyed over arbitrarily large or small portions of the field and the procedure lends itself better to physical interpretation and ease of understanding of the flow processes. Crowe and Pratt extended the tank and tube formulation for particle laden flows to include particle momentum and showed that it circumvented the continuum problem associated with the differential formulation of the equations for two-phase flows. The tank itself is the elemental volume through which the particles pass and for which the governing equations in finite-difference form must be satisfied (Fig 2). If a particle passing through the tank is accelerated in the x-direction by the aerodynamic drag of the fluid then there has to be a corresponding deficit of x-momentum of the fluid in the tank and, according to the tank and tube assumptions, is distributed uniformly throughout the tank. In essence, this approach by Crowe and Pratt has become the particle-source-in-cell (PSI-cell) method of Washington State University, USA,

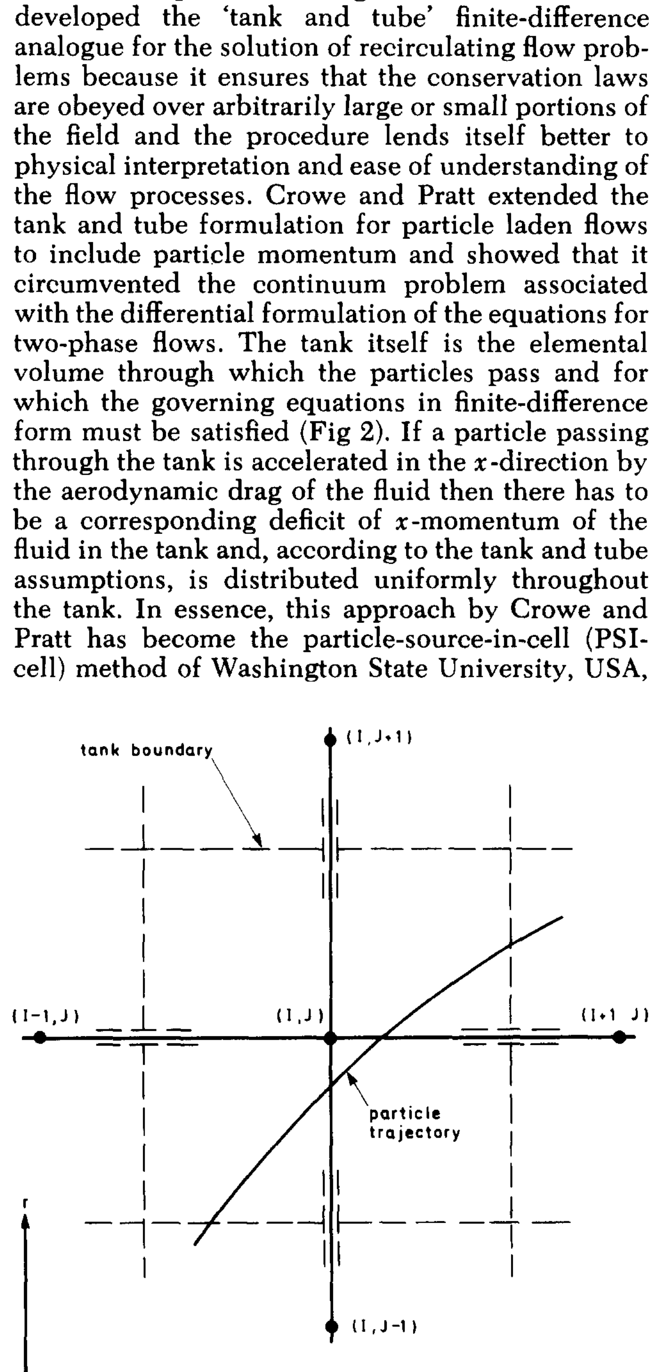

Fig 2 Tank-and-tube configuration enclosing node (I, J) (from Crowe and Pratt[8])

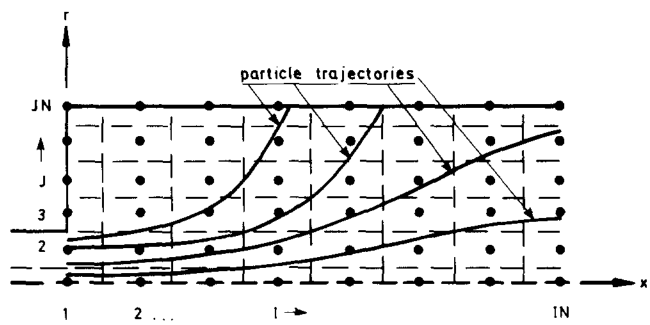

Fig 3 Particle trajectories and node distribution to analyse gas particle flow into a sudden expansion[8]

for gas-droplet flows reported in Crowe *et al*[9] and recently used to predict gas-particle flow by Lee and Crowe[37] and Sharma and Crowe[38]. Crowe and Pratt assumed that particles occupy negligible volume and their presence does not affect viscosity. They further assumed a particle size distribution in which n_i is the number of particles of mass m_i per unit volume, N is the number of particle size classes, and the density of particle mass per unit volume is $\sum_{i=1}^{N} n_i m_i$. Thus, in polar coordinates, the momentum equations for gas-particle flow are:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r \dot{r}_f \dot{x}_f) + \frac{\partial}{\partial x}(\rho(\dot{x}_f)^2) + \sum_i n_i m_i \alpha_{xi} = -\frac{\partial p}{\partial x} + \tau_x \qquad (20)$$

in the x direction, and:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r(\dot{r}_f)^2) + \frac{\partial}{\partial x}(\rho \dot{r}_f \dot{x}_f) - \rho\frac{t_f}{r} + \sum_i n_i m_i \alpha_{ri} = -\frac{\partial p}{\partial r} + \tau_r \qquad (21)$$

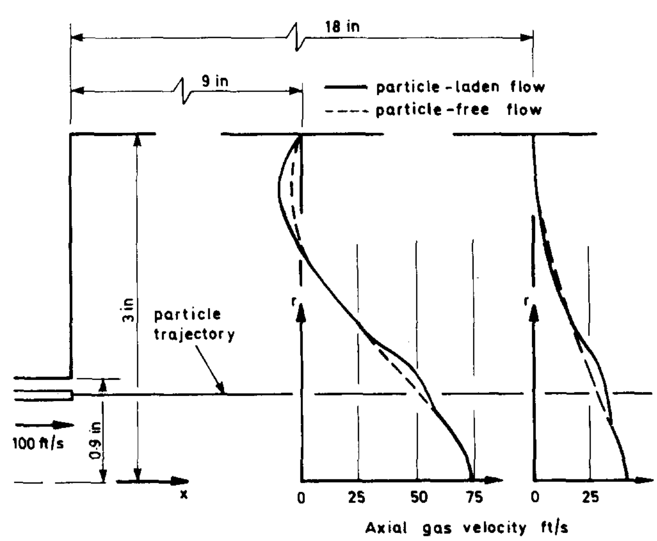

Fig 4 Effect of a particle laden stream on the axial gas velocity distribution in a non-swirling confined jet for a particle–gas mass flow ratio of 0.3 and 100 μm particles. The distributions are plotted 9 and 18 in (225 and 450 mm) downstream of the entry port (from Crowe and Pratt[8])

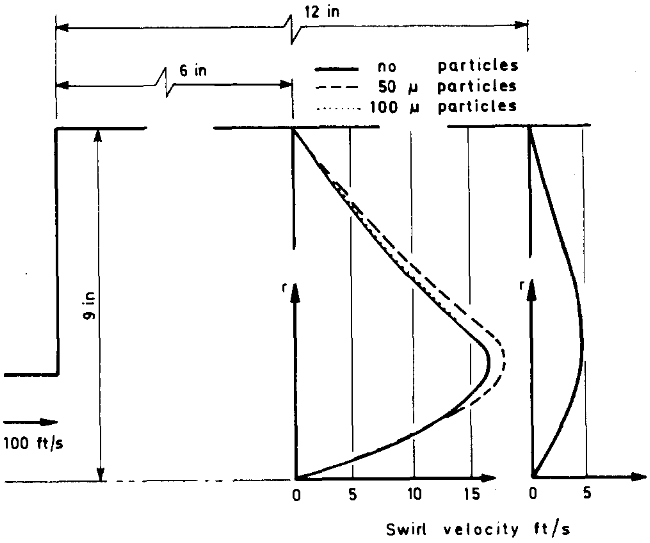

Fig 5 Effect of particles on the swirl gas velocity distribution of a confined jet with a swirl factor of unity and a particle–gas mass flow ratio of 0.3. The distributions are plotted 6 and 12 in (150 and 300 mm) downstream of the entry port (Crowe and Pratt[8])

in the radial r direction, where α_{xi} and α_{ri} are ith particle class accelerations, τ_x and τ_r the shear stresses in the x and r directions respectively, and $\dot{r}_f$ and $\dot{t}_f$ are the radial and tangential fluid velocities respectively. The particle accelerations are given by:

$$\alpha_{xi} = \frac{d\dot{x}_i}{dt} \tag{22}$$

and

$$\alpha_{ri} = \frac{d\dot{r}_i}{dt} - \frac{(\dot{t}_i)^2}{r} \tag{23}$$

Crowe and Pratt started their solution by establishing the flow field in the absence of particles. Using this flow field and an appropriate expression for drag coefficient, Eq (17) for particle motion is integrated to locate the particle trajectories in the flow. The number density of the ith particle size class is determined from:

$$n_i = \frac{\dot{\eta}_i \, \Delta t}{V_{tank}} \tag{24}$$

where $\dot{\eta}_i$ is the number flow rate of the ith particle size class along the trajectory and Δt is the residence time of a particle in the tank which is of volume V_{tank}. The flow field is recalculated incorporating the two-phase source terms in Eqs (20) and (21) from which new particle trajectories are calculated, the source terms are corrected and so on until further repetition makes no change to the solution. The predictions of Crowe and Pratt for the sudden expansion of a particle-laden subsonic swirling gas stream are illustrated in Figs 3–5. It should be noted that the tank-and-tube formulation can be extended to allow for energy coupling between phases.

In order to establish which regions of a gas turbine are most prone to erosion damage by gas borne particles, three-dimensional particle trajectory calculations are necessary. Lord and Singh[31] considered the trajectories of individual particles through an axial turbine stage assuming that concentrations were low enough for there to be no modification of the gas flow due to the presence of the particles. The particle trajectory calculations were fully three dimensional though to save computer time Lord and Singh really only considered two-dimensional gas flow; they chose the meridional plane of a blade channel. Fluid velocity distribution through a blade channel was determined using the streamline curvature cascade flow solution method of Bindon and Carmichael[39]. The particle equations of motion allowed for axes of rotation but in their chosen examples only stator blade channels were considered, and although the fluid flow programme could predict boundary-layer development, this was ignored when predicting particle trajectories. Allowance was made for angularity of the particles by a modified drag coefficient (see Eqs (9) and (10)) and Stokes number $St' = (\rho \dot{x}_f d^2 / 18 \mu_f c \lambda)$ where c is the blade axial chord and μ_f the fluid viscosity. Typical particle trajectories from Lord and Singh are shown in Figs. 6–8. They have also been concerned with the modelling of engine conditions in cascade experiments and have shown from particle trajectory calculations that errors occur when scaling only on Stokes and Mach numbers[40]. The errors are due to Reynolds number effects which cannot be eliminated, as it is impossible to simultaneously match gas and particle Reynolds numbers as well as Stokes and Mach numbers.

Abdel Azim and Rashed[34] investigated particle trajectories and their associated velocity history in centrifugal compressors. As with Lord and Singh[31], Abdel Azim and Rashed assumed that the particles had no influence on the gas flow field and that the forces on the particles were their own inertia and viscous drag. The differences between the work of Abdel Azim and Rashed and that of Lord and

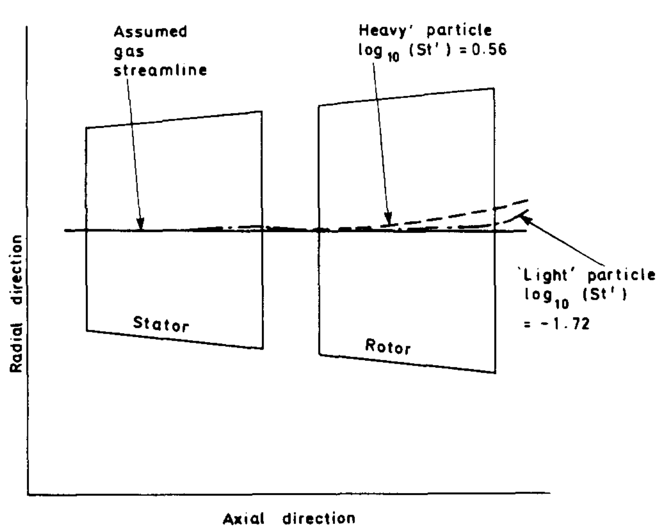

Fig 6 Calculated particle trajectories in gas turbine first stage flow path shown in meridional plane (Lord and Singh[31])

Singh, apart from the obvious centrifugal versus axial, was that the former introduced rebound characteristics for their spherical particles when impacting with compressor surfaces, whereas the latter were interested in determining if their angular particles impacted with surfaces but not if they rebounded. Abdel Azim and Rashed used Stokes law for the Reynolds number range $0 < Re < 0.1$ and the modified Stokes law for the drag force for $Re > 0.1$ (see Eq (14)). They used the rebound characteristics of Grant *et al*[41] given by:

Normal Restitution Coefficient

$$= 0.993 - 1.76\beta_1 + 1.56\beta_1^2 - 0.49\beta_1^3 \quad (25)$$

Tangential Restitution Coefficient

$$= 0.988 - 1.66\beta_1 + 2.11\beta_1^2 - 0.67\beta_1^3 \quad (26)$$

where β_1 is the relative impingement angle. Like many other workers, they solved the particle-motion equations by using a fourth order Runge–Kutta method for each particle. For gas flow analysis, the meridional plane solution was obtained by using the streamline curvature method suggested by Katsanis[42] and modified by Vanco[43]. In general this study showed that the deviation of solid particle paths from the gas streamlines increases with increased particle mean diameter and material density; the former has the greatest effect and the latter, like initial velocity, have less effect on particle dynamic behaviour.

Today, probably the most concentrated efforts in predicting three-dimensional particle trajectories in turbomachinery and understanding the influences on and effects of the particle trajectories are those at the University of Cincinnati under the direction of W. Tabakoff[33,44–54]. The work at Cincinnati is reviewed in Tabakoff and Hamed[33]. They concluded that the equations governing the motion of the solid particles in the gas flow field must be used in the most general form applicable to either fixed or rotating frames of reference. Empirical correlations for restitution ratios such as:

$$\frac{V_2}{V_1} = 1 - 2.03\beta_1 + 3.32\beta_1^2 - 2.24\beta_1^3 - 0.472\beta_1^4 \quad (27)$$

and

$$\frac{\beta_2}{\beta_1} = 1 + 0.409\beta_1 - 2.52\beta_1^2 + 2.19\beta_1^3 - 0.531\beta_1^4 \quad (28)$$

for 200 micron quartz particles impacting aluminium 2024 metal surfaces where V_2/V_1 is the relative velocity ratio and β_2/β_1 the ratio of particle trajectory to surface angle after to before impaction, have to be used. Tabakoff and Hamed determined the trajectories of individual particles assuming that drag and centrifugal forces only influenced the trajectories and using assumptions similar to those of other workers on the influence of partical trajectories, etc, on gas flow properties. The existing code of Katsanis[42] for turbomachinery flow field calculations was used to obtain the relative gas velocity components and densities of compressible non-viscous steady state gas flow at the grid points on an orthogonal mesh in the blade-to-blade surface. Linear interpolation was used to compute properties needed for the trajectory calculations at other points. The equations of motion of the solid particles used Stokes law for Reynolds numbers less than unity and the modified Stokes law for all other Reynolds

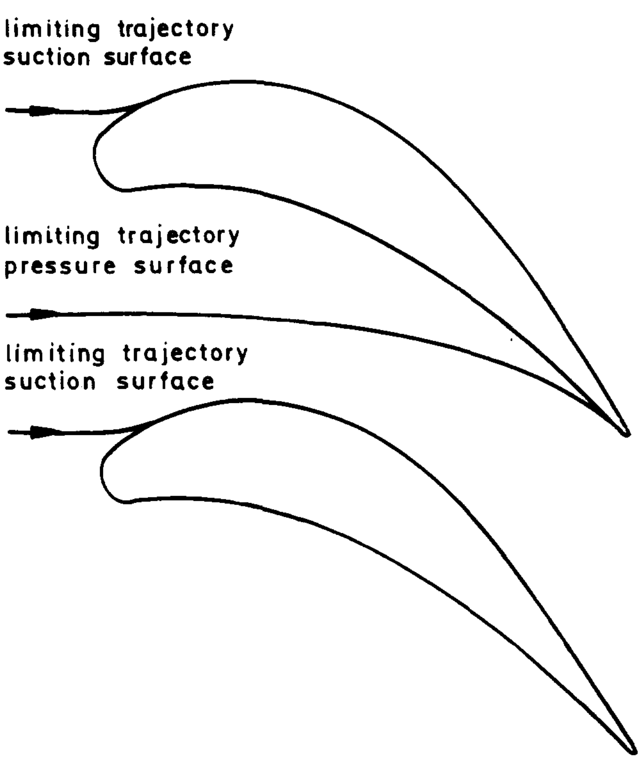

Fig 7 Impact fraction (Lord and Singh[31])

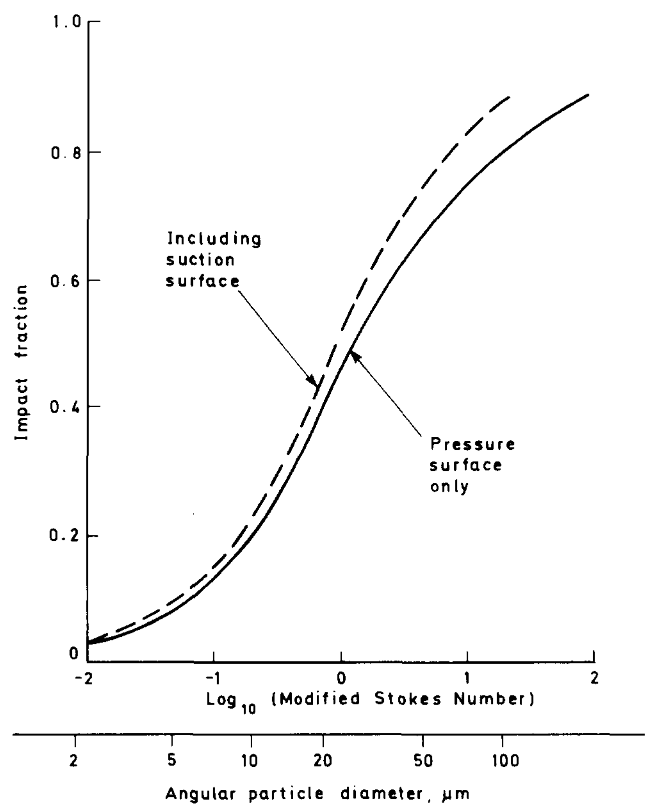

Fig 8 Calculated impact fraction for a gas turbine first stage stator at mid-height, zero initial slip and deviation[31]

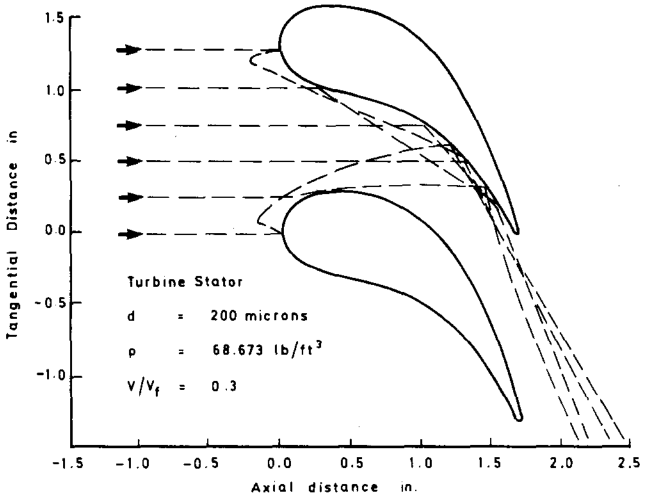

Fig 9 Particle trajectories through a turbine stator (Tabakoff and Hamed[33])

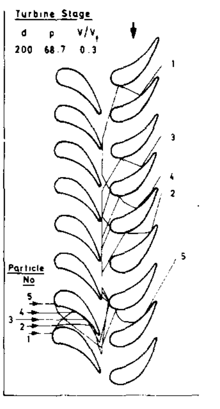

Fig 10 Axial and tangential components of particle trajectories relative to rotor blades[33]

numbers (see Eqs (11)–(13)). Typical particle trajectories are shown in Figs 9 and 10.

Concluding remarks

In the analyses discussed above, many particle-gas characteristics are taken into account but there are exceptions such as particle–particle reactions, loading ratios, lift and spin, and turbulence. The PSI-cell method takes account of loading ratio by allowing a decrease in gas momentum to offset an increase in particle momentum while passing through a cell, but others do not. If required, loading ratio could be accounted for in the other analyses, but reductions in computer time are achieved if the effects of loading ratios and the related particle–particle reactions are ignored. Justification for these omissions comes from experiment and approximate calculation methods. Rudinger[28] suggested that particle–particle reactions can be ignored if particle pitch-to-diameter ratios are greater than ten and the particles constitute less than half the mass of the mixture. Tabakoff and Hussein[45] show about 12% and 24% change in particle and gas velocities respectively at a given position in a stator blade passage for a change in loading ratio from 0.05 to 0.2; the gas velocities for a loading ratio of 0.05 are marginally different from the zero loading ratio case. For 100 μm diameter particles and particle to gas density ratio of 5000, loading ratios of 0.05 and 0.2 correspond to particle pitch-to-diameter ratios of about 35 and 22 respectively. Accepting Rudinger's observation, then the variations of Tabakoff and Hussein are due to momentum coupling between particle and gas and are not influenced by particle–particle reactions.

Some authors say they have ignored the influence of particles on viscosity, but there cannot be such an influence. Particle loadings can affect gas pressure and density and, therefore, kinematic viscosity leading to changes in Reynolds number, drag coefficient, etc, but not absolute viscosity. If, however, the analysis considers the mixture as a continuum, an effective viscosity must be used when considering shear stresses at flow boundaries and, clearly, loading ratio affects the value of effective viscosity. If this is done particle rebound characteristics play no part in the analysis.

The authors of this article are particularly interested in erosion damage of gas turbine components and, therefore, are most interested in particle trajectories including rebound characteristics. Thus the analyses of Lord and Singh[31], Abdel Azim and Rashed[34] and those of the Tabakoff team are of most interest; all these basically determine the trajectory of an individual particle ignoring its effect on the gas. This limitation common to the individual particle trajectory analyses could be overcome by analysing the trajectories of a range of individual particles through the chosen flow passage of the machine of interest on a time averaged, step-by-step basis assuming a statistical sample of sizes, shapes, velocities and directions at entry to the flow passage. Then, as in the PSI-cell method, a momentum balance between gas and particles could be undertaken throughout the flow field to determine a new gas velocity distribution which could be used to re-determine particle trajectories and so forth. This would prove costly in computer-time and, bearing in mind the findings of Rudinger[28] and Tabakoff and Hussein[45] and that loading ratios in excess of 0.2 are not very realistic, may be unnecessary and other factors could be more important. Before leaving the particle trajectory analyses, a word of caution may not go amiss. Lord and Singh[40] were concerned about modelling engine conditions in cascade experiments but, in so doing, although admitting it is not possible to simultaneously match gas and particle Reynolds numbers as well as Stokes and Mach numbers, omitted to model Froude number, which is of major consequence in view of the thousands of 'g' experienced by turbine rotors. Whatever simplifications are made in order to facilitate solution of the mathematical model they should not remove from the analyses first-order terms.

None of the analyses discussed above have taken account of free-stream turbulence. Consideration can be given to turbulence by assuming simple relationships for the gas velocities as functions of time and, therefore, space. Hinze[55] suggested assuming a coordinate system moving with the discrete particle, but this may not be possible in a turbine stage where for steady flow it is convenient to have stationary and moving frames of reference for stator and rotor blade passages respectively. Hinze noted that, when considering turbulence, apart from introducing a turbulent viscosity it is also necessary to introduce the 'Basset' term into the right hand side of Eq (15) and subsequent equations of motion. The Basset term takes account of deviations from steady flow and increases the instantaneous flow resistance to many times the steady-state value if the particle experiences high rates of acceleration due to strong external forces.

The authors of this article have developed a computer programme to determine particle trajectories for two or three-dimensional geometries, in compressible or incompressible steady-state gas flows in an axial turbine stage. The input to the programme is blade geometry and the physical properties of the particles and gas. The output includes streamline coordinates, gas velocity, magnitude and direction throughout the blade passages, the blade surface velocities and the particle trajectory and velocity throughout the blade passage giving the locations of collision between particle and blade and casing surfaces. Account is taken of particle dynamics including lift and spin and entry to a stage other than on a streamline. The results of the programme will be published in a subsequent article.

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BOOK REVIEW

Heat exchange design handbook

This 2080 page handbook consists of five volumes, each volume being devoted to a different aspect of heat exchanger design. Volumes one and two are concerned with theoretical aspects of thermal and hydraulic design; experimental data are presented, together with recommended correlations. Volume three indicates how the material should be used in solving heat exchanger design problems; most types of equipment are discussed but most space is given to shell and tube exchangers. Volume four is devoted to mechanical aspects of design and volume five gives simple rules for estimating physical properties of fluids for which data is sparse, together with tables of properties of the more widely used substances.

More than sixty authors have made contributions to the handbook, and a different editor has been responsible for each volume. This has led to some repetition, and a wide variety of styles.

The stated aims were to collect heat transfer information to create a comprehensive data base which could be used for all heat exchanger designs. The book goes a long way towards achieving these aims, and will be a worthwhile addition to the library of any firm engaged in practical heat transfer design. It is not possible, however, to pick up the book and solve problems quickly. Although recommended correlations are given in some chapters, it would be useful to give them for all subjects, and to highlight the recommended correlations at the end of each chapter. Some design examples are given, but it is considered that one comprehensive example, giving a design procedure, together with reasons for choice of exchanger type, and materials used, would place the book more firmly in the design handbook category.

Some of the nomenclature will be strange to the old established designer, but, providing it is to be a universal standard, the designer will become familiar with it. It is not a good idea, however, to print the same list at the beginning of each volume, and this is particularly the case with respect to the volume on mechanical design. It would be more convenient if symbols were placed in alphabetical order, and the list of basic quantities should surely include temperature and pressure.

Since new information is continually being produced, an updating procedure has been established; the handbook has therefore been produced in a loose-leaf, ring binder format to facilitate this useful service.

N. Marshall
NEI Nuclear Systems Ltd, UK

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